EUROPEAN JOURNAL OF MATERIALS SCIENCE AND ENGINEERING
Volume 3, Issue 1, 2018: 9-24 | www.ejmse.tuiasi.ro| ISSN: 2537-4338

AN APPROACH TO CUSTOMIZABLE OPTIMALLY PROPORTIONED HIGH PERFORMANCE CONCRETE MIXTURES

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Abstract

'Smart' cement use in HPC mixture design that simultaneously enhances performance and sustainability has been a long-sought goal of cement concrete mixture researchers. In the present concrete mixture-response modeling study, proposed 3-parameter exponential mixture-response models were fitted to available test-performance data sets of HPC mixtures proportioned based on the best combined grade-aggregates (minimum void) to generate mixture-strength and mixture-porosity development (age-strength or porosity) profiles of HPC mixtures and deemed robust enough to yield reliable determination of rate-parameters S_o, S_p, S_i and P_o, P_p, P_i as functions of mixture-factors that permitted reliable quantification of contributions to HPC mixture performance of individual mixture-factors and optimization of mixture properties under study (over the study domain). Mixture-response trace plot (RTP) or sensitivity analysis functions were developed to construct mixture-factor envelopes that allowed optimal tailoring of HPC mixture requirements to HPC mixture performance and an important aid in designing efficient cement concrete mixtures and effectively utilizing their unique and distinctive material properties for widened and extended practical applications. The study results validated an effective HPC mixture optimization approach allowing optimized trade-offs between mutually exclusive requirements for performance (workability, strength, durability) and sustainability (the economic and efficient use of materials) and customization of available HPC formulations.

Keywords: *mixture, performance, sustainability, model, matrix, optimization*

Introduction

High Performance Concretes

The American Concrete Institute [1] defines high performance concrete (HPC) as concrete meeting special combination sof performance, durability and uniformity requirements that can not always be achieved customarily using conventional constituents and production practices. It is basically constituted of the same materials as conventional (normal) concrete but also incorporates supplementary cementitious materials (SCM), high-performance admixtures and steel micro-fibres to obtain the required properties. This family of cement concretes comprises of, on the basis of mixture-strength, high-strength (≥50-90 MPa), very high-strength (≥90-130 MPa) and ultra-high-strength (≥130-200 MPa) concrete (modified from Büyüköztürk et al [2]).

Normal strength concrete structures have a mass to strength ratio of 40-120 kg/MNm while that for HPC structures averages 15 kg/MNm [3] but HPC mixtures have the disadvantage of high and expensive contents of cement, chemical additives, (if needed) micro-steel fibres in addition to having a higher carbon footprint and although, an increasing range of HPC formulations is available and can be adjusted to meet the specific requirements of an individual design, construction or

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architectural description, the dominant HPC products on the market are proprietary and expensive. The information on their compositions is not readily available and almost impossible to modify or customize the proportions for specific application.

Non-optimized HPC mixtures are almost always uneconomical and inefficient but only optimization of HPC mixtures on a quantitative basis can yield efficient HPC mixtures with cement content balanced to achieve the desired performance while minimizing the risk of problems arising from high cement content (the obvious increased cost, the negative environmental effects, shrinkage and cracking problems) and reduced resource requirements in infrastructural applications. The traditional method for optimizing HPC mixtures to achieve the desired performance (which involves systematically varying individual mixture-factors in small increments and studying the resultant effect) is time-consuming, requiring a large number of trial batches and hence expensive and inefficient. In this method, the basis for selecting SCM dosage is arbitrary and often focuses on a specific set of requirements such as strength or durability. The use of the proposed mixture-factor envelopes has the potential to reduce the number of test runs needed, especially when multiple cementitious components are used and multiple requirements have to be simultaneously satisfied.

Cement Concrete Mixture Matrix Structure

Nearly a century after Abrams [4] proposed the water-to-binder ratio law, much has been contributed by cement concrete mixture researchers to broaden the understanding of how the fresh and hardened-state properties of concrete are controlled by the relative proportions of concrete constituent components—cement, coarse and fine aggregates, water, and various additives—while elevating concrete to the status of the most dominant construction material for 21st century infrastructural needs. Important advances in admixture technology over the past decades and a recognition that coarse aggregates represent the weakest link in concrete and that they can be taken out to have only sand as the main aggregate (Fig. 1) have led to the development of a new generation of cement concrete mixtures with low water-binder ratio, low matrix porosity and high particle packing density that lead to far improved rheological, mechanical and durability properties than obtains with conventional cement concretes (CCC) at a similar unit weight [5]. Such cement concrete mixtures (collectively termed as high-performance concretes or engineered ‘high-tech’ concretes designed to meet project-specific needs) incorporate fine-grained additives (fga) and high-range water reducers (see Fig. 1).

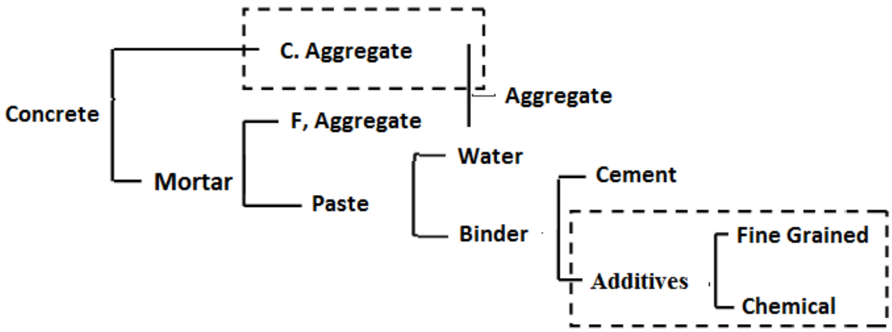

Fig. 1. Cement Concrete Mixture Matrix Structure

The incorporation of fine-grained additives produces a wide and continuous grain size distribution that helps in optimizing packing density of the matrix, creates a more uniform stress distribution when the matrix is loaded and hence a strong matrix; the smaller grains serve as a lubricant that reduces the inter-particle friction and hence improved workability of the mixture; the fine-grains also lower the porosity of the system by filling the voids of the mixture with fine particles while expelling water from the voids to allow the water to be more homogeneously distributed in the system and hence, again, improve the workability of the mixture and produce a strong mixture matrix; the super-plasticizer helps disperse cement and filler particles, improve the lubrication and reduce the inter-particle friction and improve the workability of the mixture [5].

Nevertheless, predicting common time-dependent mechanical behaviours of cement concretes remains inherently complex because of the heterogeneous nature of cement concrete, compelling many HPC mixtures to be proportioned based on prior history of production (historical information or experience) or the cost-prohibitive trial mixtures (trial-and-error) or by prescribing the limits (maximum or minimum) of the mixture-factors that circumscribe the desired performance indicators seldom based on the actual needs of the mixture and the locally available materials and do not often involve properly balancing mutually exclusive requirements for performance (workability, strength, durability) and sustainability (the economic and efficient use of materials) [6, 7, 8]. Studies have shown that prescription-based design approaches do not always provide the desired end results, because they tend to deliberately promote overdesigned mixtures by using cement content as a safety factor [8] while the trial-and-error modification of existing HPC recipes although very popular for HPC mixture design in the HPC production sector is a hit-or-miss affair, expensive, wasteful and deemed inoptimizable because of the many varied material-inputs involved and the different sources for the material-inputs [9].

Cement Concrete Mixture Performance Characterization

Optimized design of HPC mixtures involves consideration of more varied material constituents (and potentially more interactions among the material constituents), mixture proportions and properties, all optimized to determine the most economical and practical mixture-material constituent quantities to produce concrete of desired fresh-state properties (workability, pumpability, finishability, and consistency) and required hardened state properties (strength and durability-related properties such as water-tightness, wear and sulphate resistance, etc) consistent with particular conditions of use. Current mixture design of HPC production focuses on optimizing mixture properties (workability, strength and durability) of concrete in fresh and hardened states by optimizing the particle packing density of the granular ingredients of HPC as the accepted key mixture design consideration although the most common industry practice is to simply modify, by trial and error, existing HPC mixture recipes [10, 11]. Sabir [12] has defined HPCs as cement concrete mixtures in which each granular ingredient performs effectively to contribute towards the HPC mixture's fresh and hardened state properties. The focus of this work is the quantification of the contribution of each HPC mixture-factor towards an HPC mixture's fresh and hardened state properties and the optimization of these contributions to allow optimal tailoring of HPC mixture requirements to HPC mixture performance to achieve efficiency and economy in HPC mixture design using cold-cast HPC mixture test-performance data sets available in cement concrete mixture research literature but also to allow modification or customization of available high performance concrete (HPC) formulations to meet specific infrastructural applications.

Methods

Parameterized Mixture-Strength and Mixture-Porosity Response Models

One-hundred thirty-four (134) sets of test-data and test-results for HPC mixtures from available cement concrete mixture research literature were employed to construct parameterized mixture-strength and mixture-porosity response models following the determination of model rate-parameters S_o, S_p, S_i and P_o, P_p, P_i. Cement concrete mixture-composition optimization models were then built using the developed parameterized mixture-strength and mixture-porosity models and constructed mixture-factor envelopes.

Proposed Parameterized Mixture-Strength and Mixture-Porosity Response Models

Porosity and compressive strength of cement concrete mixtures are interconnected and directly influence the failure behaviour of cement concretes. The time-dependent mechanical behaviour of concrete mixtures are characterized by an inner interval with early accelerated activity and a later outer interval with responses stabilized to a constant rate of increase which suggests a multiple-time scale problem and amenable to matching-approximations analysis [13]. The following uniform composite approximations are offered for the general models for estimating concrete mixture-strength S_t and concrete mixture-porosity P_t after t days of curing:

$$S_t = [S_o + S_p t][1 - e^{-(S_i/S_o)t}] \tag{1}$$

$$P_t = [P_o - P_p t]/[1 - e^{-(P_i/P_o)t^{1/2}}] \tag{2}$$

where:

S_o, S_p, S_i and P_o, P_p, P_i are mixture-strength and mixture-porosity rate-parameters, respectively, that are functions of mixture-factors of the cement concrete mixture-matrix structure properties (see Fig. 1).

The proposed models only require a determinations of the rate-parameters S_o, S_p, S_i and P_o , P_p, P_i from available test data or their evaluation through prediction models based mixture-factors screened from the cement concrete mixture-matrix structure properties.

Proposed Mixture-ResponseRate-Parameter Models

The aggregative effect of the enormous number of cement concrete mixture-constituents and properties contributing to the mixture-strength and mixture-porosity (and other desired concrete attributes) can be captured through the three rate-parameters as follows:

$$S_o = \prod_{j=1}^{m} \pi_j^{\alpha_{oj}} \; ; \quad S_p = \prod_{j=1}^{m} \pi_j^{\alpha_{pj}} \; ; \quad S_i = \prod_{j=1}^{m} \pi_j^{\alpha_{ij}} \tag{3}$$

$$P_o = \prod_{j=1}^{m} \pi_j^{\beta_{oj}} \; ; \quad P_p = \prod_{j=1}^{m} \pi_j^{\beta_{pj}} \; ; \quad P_i = \prod_{j=1}^{m} \pi_j^{\beta_{ij}} \tag{4}$$

where m is the number of independent parameterized mixture variables considered, π_j is the j^{th} independent parameterized mixture variable; α_{oj} , α_{pj} , α_{ij} and β_{oj} , β_{pj} , β_{ij} are the exponents to be determined through regression analysis. Independent parameterized mixture variables screened from the cement concrete mixture-matrix structure (see Fig. 1) include:

π_1 : RHA surface area factor, $\frac{100 + \%\,\mathrm{rha}_{SA}}{100}$

π_2 : water content factor, kg/cu. m

π_3 : cement content factor, kg/cu. m

π_4 : silica-fume factor, $\frac{100 + \%\mathrm{sf}}{100}$

π_5 : fga factor, $\frac{100 + \%\,\mathrm{fga}\,/[\mathrm{w/b}\ +1]}{100}$

π_6 : super-plasticizer content factor, kg/cu. m

π_7 : coarse aggregate-to-binder ratio

π_8 : water-to-binder ratio factor, $\frac{\mathrm{w}}{\mathrm{b}}$

π_9: entrapped air factor, $\frac{100 + \%\ \mathrm{Entrapped\ \ Air}}{100}$

π_{10}: mixture-slump factor, mm

π_{11} sand percentage in the mixture-aggregate, %

π_{12}: sand packing factor, $\frac{\alpha_{\mathrm{cement}} + \alpha_{\mathrm{sand}}}{\alpha_{\mathrm{cement}}}$

π_{13}: RHA packing factor, $\frac{\alpha_{\mathrm{cement}} + \alpha_{\mathrm{RHA}}}{\alpha_{\mathrm{cement}}}$

π_{14} silica-fume packing factor, $\frac{\alpha_{\mathrm{cement}} + \alpha_{\mathrm{SF}}}{\alpha_{\mathrm{cement}}}$

Mixture-Response Model Calibrations

HPC mixture test-data from tests by Azizinamini [14], Domone and Soutsos [15] and Nguyen [16] were used to calibrate the proposed mixture-response models. MATLAB®'s non-linear least-squares (nlinfit) regression analysis and MICROSOFT EXCEL®'s linear least squares (linest) regression analysis programmes were employed to fit concrete test-data to the proposed parameterized mixture-strength and mixture-porosity response models and to perform rate-parametric analyses to obtain model coefficients, respectively, as follows:

Mixture-Strength Response Rate-Parameters

$$S_o = \pi_1^{\ 0.06}\pi_2^{\ 0.11}\pi_3^{\ 0.77}\pi_4^{\ 1.81}\pi_5^{\ 0.39}\pi_6^{\ 0.00}\pi_7^{\ 1.06}\pi_8^{-0.61}\pi_9^{\ -3.28}\pi_{10}^{\ 0.29}\pi_{11}^{\ 0.23} \tag{5}$$

$$S_p = \pi_1^{\ -0.03}\pi_2^{\ -0.51}\pi_3^{\ -1.16}\pi_4^{\ -4.48}\pi_5^{\ 2.56}\pi_6^{\ -0.03}\pi_7^{\ -2.06}\pi_8^{\ -1.55}\pi_9^{\ 3.91}\pi_{10}^{\ -1.49}\pi_{11}^{\ 1.32} \tag{6}$$

$$S_i = \pi_1^{\ 0.02}\pi_2^{\ 0.41}\pi_3^{\ -0.23}\pi_4^{\ -2.85}\pi_5^{\ 0.43}\pi_6^{\ -0.02}\pi_7^{\ 0..86}\pi_8^{\ -2.06}\pi_9^{\ 2.15}\pi_{10}^{\ -0.79}\pi_{11}^{\ 1.27} \tag{7}$$

Mixture-Porosity Response Rate-Parameters

$$P_o = \pi_3^{\ 0.76}\pi_4^{\ -1.33}\pi_6^{\ -4.87}\pi_8^{\ -0.04}\pi_{11}^{\ 0.01}\pi_{12}^{\ -0.32}\pi_{13}^{\ 1.03}\pi_{14}^{\ 1.48} \tag{8}$$

$$P_p = \pi_3^{\ 28.7}\pi_4^{\ -29.5}\pi_6^{\ -31.1}\pi_8^{\ 13.9}\pi_{11}^{\ -3.66}\pi_{12}^{\ -31.1}\pi_{13}^{\ 12.2}\pi_{14}^{\ 25.9} \tag{9}$$

$$P_i = \pi_3^{\ 2.11}\pi_4^{\ -2.75}\pi_6^{\ -11.9}\pi_8^{\ -0.03}\pi_{11}^{\ 0.14}\pi_{12}^{\ -2.26}\pi_{13}^{\ 1.47}\pi_{14}^{\ 1.74} \tag{10}$$

Predicted results using the developed parameterized mixture-strength and mixture-porosity response models and those predicted by mixture-strength response models proposed by Sarkar et al. [17] and Rajasekaran [18] are shown in Table 1, Table 2, Table 3 and Table 4.

Table 1. Mixture-Data, Test- and Model-Results for High Strength HPC Mixtures

Source	Mixture ID	$\frac{w}{b}$	C. aggr. kg/cu. m	Water kg/cu. m	Cement kg/cu. m	FA kg/cu. m	SF kg/cu. m	SP kg/cu.m	Entrained Air %	Slump mm	*Test Result MPa	Model Result MPa
Azizinamini [14]	MS1‡	0.228	655.6	137.9	484.4	85.7	33.9	15.1	2.3	260	68.3	76.6
	MS3‡	0.241	623.6	163.0	550.2	87.0	38.6	10.1	1.5	267	64.1	71.9
	MS13‡	0.282	625.9	150.5	449.8	52.9	31.4	13.4	7.0	267	66.0	59.2
	MS12‡	0.248	596.9	170.3	537.9	85.1	64.7	10.3	2.1	279	71.7	69.9
	T1	0.357	685.9	155.1	328.5	82.2	23.1	2.7	5.4	89	51.4	56.4
	T3	0.265	668.1	130.1	382.6	82.1	26.6	7.4	7.4	267	67.1	63.6
	T4	0.269	747.0	112.5	338.8	56.4	23.9	6.3	4.5	222	70.7	74.6
	T5‡	0.359	680.5	169.6	390.2	55.7	27.1	3.0	2.2	152	58.2	57.3
	T6	0.294	697.1	134.0	360.7	69.5	25.5	4.8	5.4	140	64.8	62.2
	T7‡	0.400	709.0	163.0	329.6	54.9	23.3	2.5	3.5	140	50.4	53.6
	T8	0.309	686.4	138.8	355.3	68.4	25.1	4.8	6.4	178	60.7	58.5
	T10‡	0.280	675.8	116.3	314.9	78.8	22.2	6.2	10.3	235	58.1	58.2
	T11‡	0.364	646.7	178.8	382.1	82.1	26.5	3.1	5.1	114	51.4	51.5
	T12	0.301	666.9	131.2	345.1	66.4	24.4	4.6	8.5	229	61.4	57.0
	T14‡	0.293	723.8	139.1	374.6	72.1	28.0	5.0	6.7	216	66.2	63.5
	T15‡	0.300	650.9	133.3	341.9	79.1	24.1	4.7	10.0	229	55.6	54.1
	T17	0.317	716.7	135.5	333.9	69.6	23.6	4.5	5.1	191	59.8	61.3
	T18	0.345	672.8	154.0	353.4	68.1	25.0	2.8	6.6	165	52.7	53.0
	MS2	0.229	656.8	144.1	485.7	85.9	58.1	9.4	1.8	267	83.8	79.9
	MS4	0.233	677.5	134.9	486.9	57.3	34.1	8.7	2.7	229	82.6	76.7
	MS5	0.231	644.3	147.9	518.2	72.0	49.2	12.8	1.9	267	77.6	77.3
	MS6	0.229	644.9	147.0	546.5	57.4	38.3	16.1	NR	254	71.4	-
	MS7	0.226	662.7	136.7	488.2	57.4	58.4	15.1	2.5	267	88.0	81.4
	MS8	0.223	631.3	151.1	551.9	58.0	66.4	10.1	1.9	241	84.1	78.2
	MS9	0.219	606.4	152.9	546.8	86.5	65.7	17.5	2.8	267	83.9	75.2
	MS10	0.234	640.8	148.6	515.5	71.7	47.8	12.7	2.3	267	81.5	75.0
	MS11	0.232	620.6	154.4	542.5	57.0	65.3	16.6	2.3	267	80.6	73.9
	MS14	0.229	662.7	138.2	488.5	57.4	58.5	9.1	2.3	229	86.9	78.5
	MS15	0.212	653.8	137.6	525.8	73.1	50.0	13.0	2.3	229	83.6	82.3
	MS16	0.221	655.0	144.4	555.1	58.3	38.9	9.8	2.3	254	88.1	78.8
	MS17	0.211	630.7	142.7	551.2	87.2	38.6	16.9	2.3	267	78.5	77.6
	MS18	0.242	658.0	146.7	486.5	86.1	34.0	9.1	2.3	254	81.6	73.7
	MS19	0.233	627.7	144.2	476.8	84.4	57.1	15.5	2.3	267	76.5	72.1
	MS20	0.206	655.6	134.0	527.6	73.3	50.1	13.0	2.3	254	78.9	83.0
	MS21	0.224	642.0	145.1	512.0	87.5	49.8	13.2	2.3	248	79.5	76.6
	MS22	0.206	650.3	133.7	523.0	77.9	49.7	9.8	2.3	254	84.5	83.1
	MS23	0.238	661.5	144.3	488.2	71.9	46.6	12.1	2.3	254	79.4	77.0
	MS24	0.230	639.6	146.1	514.7	71.6	48.9	15.9	2.3	248	78.4	74.5
	MS25	0.216	641.4	138.2	519.6	71.7	49.0	12.7	2.3	254	83.4	78.9
	MS26	0.221	655.6	138.2	519.4	57.7	49.4	12.5	NR	229	84.8	-
	MS27	0.225	651.4	140.7	516.4	71.8	36.1	12.5	2.3	254	84.6	77.0
	MS28	0.223	630.1	150.3	550.6	72.5	52.2	13.5	2.3	254	76.9	76.5
	MS29	0.244	632.5	158.4	516.1	71.7	61.9	13.0	2.3	254	84.4	72.9
	MS30	0.214	651.4	138.2	523.9	72.9	49.8	12.9	2.3	229	89.0	79.6
	T2‡	0.306	706.6	141.3	365.7	70.4	25.8	4.9	2.3	152	67.9	63.3
	T6	0.294	697.1	134.0	360.7	69.5	25.5	4.8	2.3	140	64.8	62.2
	T9	0.264	709.0	126.4	394.7	56.3	27.4	7.2	2.3	229	75.2	71.7
	T13	0.276	686.4	135.5	393.4	70.3	27.3	5.2	2.3	171	71.8	64.7
	T16	0.302	711.4	138.4	363.3	70.0	25.7	6.9	2.3	203	76.5	64.3
	T19	0.267	712.6	124.2	368.9	71.0	26.0	5.0	2.3	114	76.3	70.2
	T20	0.289	716.7	129.2	365.6	56.2	25.8	4.8	2.3	203	70.2	68.1

Note: 1.0 lb/cu. yd = 0.5933 kg/m^3 1.0 in = 25.4 mm 1.0 psi = 0.006895 N/mm^3 1.0 oz/cu. yd = 0.03708 kg/m^3
‡mixture used in constructing mixture-response rate-parameter model sand content in aggregate: 54.5%
w/b: water-to-binder ratio c. aggr: coarse-aggregate fa: fly-ash sf: silica-fume sp: super-plasticizer
NR: Not Recorded *28-day mix-compressive strength

Table 2. Mixture-Data, Test- and Model-Results for Higher Strength HPC Mixtures

Source	Mixture ID	$\frac{w}{b}$	Binder kg/cu. m	Water kg/cu. m	†%Cement	†%FA	†%SF	†%GGBS	†%SP	Slump mm	*Test Result MPa	Model Result MPa	SAJ Model Result MPa	SLNN Model Result MPa
Domone and Soutsos[15]	H2‡	0.32	454	145.3	100				0.80		90.0	91.0		
	K4	0.32	454	143.5	95		5		0.89		94.5	95.3	90.1	113.5
	K5	0.32	454	143.5	90		10		0.97		106.0	99.3	91.3	117.3
	K6‡	0.32	454	143.5	85		15		1.05		107.5	102.7	95.0	120.0
	K7	0.29	492	142.7	95		5		1.08		99.5	99.2	95.1	119.0
	K8	0.29	492	142.7	90		10		1.08		114.5	103.4	96.5	122.4
	K9	0.29	492	142.7	85		15		1.08		118.5	107.2	100.3	124.5
	K10	0.26	510	132.6	95		5		1.32		110.0	104.6	100.2	122.1
	K11	0.26	510	132.6	90		10		1.22		113.5	109.2	101.7	124.5
	K12	0.26	510	132.6	85		15		1.12		115.0	113.4	105.7	125.5
	K13‡	0.23	547	125.8	95		5		1.67		111.5	110.5	105.4	128.3
	K14	0.23	547	125.8	90		10		1.67	150	116.0	115.6	107.3	132.2
	K15‡	0.23	547	125.8	85		15		1.67		125.0	120.2	111.8	134.8
	H3	0.29	492	142.7	100				1.50		96.5	94.3		119.2
	H4	0.26	510	132.6	100				2.33		110.0	99.6		128.6
	H5‡	0.23	547	125.8	100				0.46		106.5	105.1		109.5
	L3‡	0.26	510	132.6	38		5	57	0.63		93.5	96.2		
	L4	0.26	510	132.6	36		10	54	1.22		94.0	96.1		
	L5‡	0.26	510	132.6	54	36	10		1.22		90.0	110.0		
	L6‡	0.20	590	118.0	90		10		2.22		118.0	123.4	113.2	141.1
	L7‡	0.20	590	118.0	40		10	50	2.22		113.5	116.5		
	L8‡	0.20	590	118.0	60		10	30	2.22		115.0	129.4		
	L9	0.20	590	118.0	60	30	10		2.22		103.5	116.5		
	I8‡	0.26	510	132.6	80	20			0.80		94.0	108.3		
	J4‡	0.32	454	145.3	90		10		0.50		90.0	95.3	90.1	112.1
	J7	0.26	510	132.6	90		10		0.97		105.0	104.9	95.1	121.5
	J8‡	0.26	510	132.6	70		30		0.74		105.0	109.5	101.7	119.2

Note: 1.0 lb/cu. yd = 0.5933 kg/m^3 1.0 in = 25.4 mm 1.0 psi = 0.006895 N/mm^3 1.0 oz/cu. yd = 0.03708 kg/m^3
coarse aggregate = 1115 kg/m^3 fine aggregate (sand) = 670 kg/m^3
‡mixture used in constructing mixture-response rate-parameter model SAJ: Sarkar et. al [17] SLNNRajasekaran [18]
w/b: water-to-binder ratio c. aggr: coarse-aggregate fa: fly-ash sf: silica-fume ggbs: ground-granulated blast-furnance slag sp: super-plasticizer sand content in aggregate: 37.5% †%: as a percentage of binder*28-day mix-compressive strength

Table 3. Mixture-Data, Test- and Model-Results for Ultra-High Strength HPC Mixtures

Source	Mixture ID	$\frac{w}{b}$	Binder kg/cu. m	Water kg/cu. m	Cement kg/cu. m	††%RHA	††%SF	†%SP	Slump mm	*Test Result MPa	Model Result MPa
Nguyen [16]	‡SF20		1062	159.3	885		20	0.76		168	169
	SF15RHA5		1062	159.3	885	5	15	1.15		174	170
	SF10RHA10		1062	159.3	885	10	10	1.15		184	168
	SF5RHA15		1062	159.3	885	15	5	1.15		176	165
	‡RHA20(3.6)		1062	159.3	885	20		1.15		176	162
	RHA20(5.6)		1062	159.3	885	20		1.75		132	163
	‡RHA20(6.3)		1062	159.3	885	20		1.20		156	162
	RHA20(9.0)		1062	159.3	885	20		1.15		174	162
	RHA20(5.6)		1062	159.3	885	20		0.89		180	162
	‡REF		1140	205.3	1140			0.90		162	155
	‡SF20	0.18	1062	159.3	885		20	0.76	210-230	164	169
	‡REF		1140	205.2	1140			0.90		163	155
	RHA10(5.6)		1110	200.0	1110	10		1.15		170	165
	SF10		1110	181.8	885		10	0.76		163	165
	SF20		1062	191.2	885		20	0.76		164	169
	‡RHA20(5.6)		1062	137.7	885	20		1.15		174	164
	SF30		995	191.2	765		30	0.76		142	168
	‡SF10		1110	137.7	1010		10	0.76		170	164
	SF10RHA10(5.6)		1062	181.8	885	10	10	1.15		185	167
	SF10RHA20(5.6)		995	137.7	765	20	10	1.15		166	161

Source	Mixture ID	$\frac{w}{b}$	Binder kg/cu. m	Water kg/cu. m	Cement kg/cu. m	††%RHA	††%SF	†%SP	Slump mm	*Test Result MPa	Model Result MPa
	‡SF10RHA30(5.6)		903	116.1	645	30	10	1.15		154	151

Note: SF(A)RHA(B)(C): A: %sf B: %rha C: rha grain size1.0 lb/cu. yd = 0.5933 kg/m^3 1.0 in = 25.4 mm 1.0 psi = 0.006895 N/mm^3 1.0 oz/cu. yd = 0.03708 kg/m^3
‡mixture used in constructing rate-parameter model rha surface area ≈ 3x(sf surface area) ≈ 62x(cement surface area)
w/b: water-to-binder ratio rha: rice-husk ash sf: silica-fume sp: super-plasticizer
sand content in aggregate: 100% †%: as a percentage of binder ††%: as a percentage of cement*28-day mix-compressive strength

Table 4. Mixture-Data, Test- and Parameterized Mixture-Porosity Model-Results for Ultra-High Strength HPC Mixtures

Source	Mixture ID	Sand%	†%w/b	Cement kg/cu. m	†%SF	†%SP	†%RHA	†Packing Density, α				‡Test Result %	Model Result %
								Cement	Sand	RHA	SF		
	REF	100	18.0	1140.0	0.0	0.9	0.0	0.399				7.5	8.67
	RHA20	100	18.0	885.0	0.0	1.15	20.0	0.399	0.478	0.364		5.76	5.87
	SF20	100	18.0	885.0	20.0	0.76	0.0	0.399			0.64	4.55	4.56
	S4	0	25.0	1140.0	0.0	0.8	0.0	0.399				10.61	9.18
	S5	0	25.0	1076.3	0.0	0.8	5.0	0.399				10.87	8.90
	S6	0	25.0	1012.5	0.0	0.8	10.0	0.399				11.68	8.91
	S7	0	25.0	885.0	0.0	0.8	20.0	0.399		0.364		8.95	8.99
Nguyen [16]	S8	0	25.0	1076.3	5.0	0.8	0.0	0.399				7.52	6.86
	S9	0	25.0	1012.5	10.0	0.8	0.0	0.399				6.23	5.22
	S10	0	25.0	885.0	20.0	0.8	0.0	0.399				4.53	2.94
	S11	0	40.0	1140.0	0.0	0.8	0.0	0.399				17.46	16.98
	S12	0	40.0	1076.3	0.0	0.8	5.0	0.399				16.18	18.88
	S13	0	40.0	1012.5	0.0	0.8	10.0	0.399				16.7	18.92
	S14	0	40.0	885.0	0.0	0.8	20.0	0.399			0.64	20.93	19.13
	S15	0	40.0	1076.3	5.0	0.8	0.0	0.399				13.27	14.81
	S16	0	40.0	1012.5	10.0	0.8	0.0	0.399				12.91	11.53
	S17	0	40.0	885.0	20.0	0.8	0.0	0.399				13.26	6.81

†Packing density, α (based on LPM—Linear Packing Model [19]) ‡Total porosity of samples measured by mercury intrusion porosimetry (MIP) at 28 days [19]

Proposed Parameterized Mixture Response Optimization Algorithm

Using the developed parameterized mixture-strength and mixture-porosity models and an optimization scheme based on a linearly weighted least-squares algorithm, mixture optimization models were constructed for mixtures investigated in the study as follows:

Defining a linearly weighted summated mixture-response (LWSMR) function

$$\mathrm{R_i} = \sum_{\mathrm{j=1,1}}^{\mathrm{N}} \mathrm{w_j}\overline{\mathrm{R}}_{\mathrm{ji}} \tag{11}$$

with individual normalized mixture-response functions

$$\overline{\mathrm{R}}_{\mathrm{i}} = \frac{1}{\mathrm{N}}\frac{\mathrm{f'_i(a)}}{\mathrm{f'_{i(ref)}(a)}} \tag{12}$$

where: $f'_i(a)$ are maxima (or minima) of mixture-response trace plot (RTP) or sensitivity analysis functions [20] shown in Fig. 2, Fig. 3, Fig. 4 and Fig. 5 to facilitate determination of the N mixture-factor weights, w_i, provided $\sum_{i=1}^{N} w_i = 1.00$ and $0 \leq w_i \leq 1.00$ by minimizing the squared sum of deviations between the weighted-values and the target value, T.

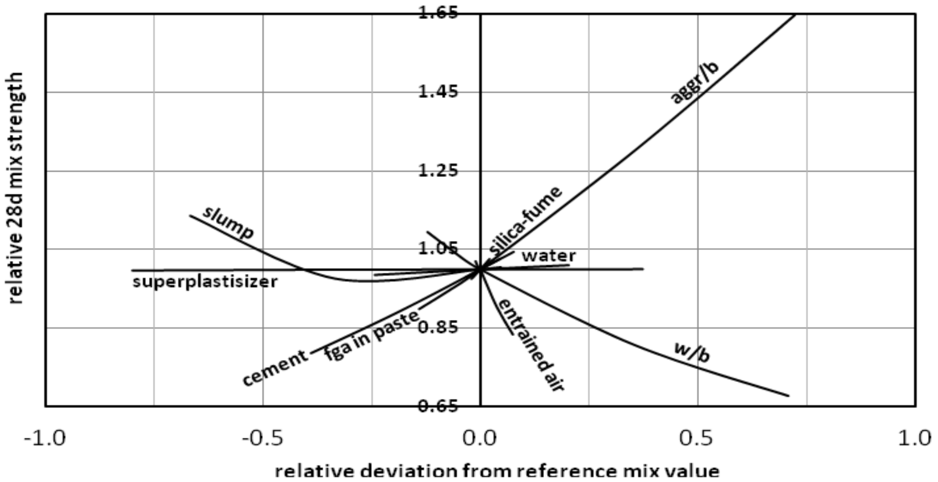

Fig. 2. Mixture-Strength Response Trace Plots of mixture M22

$$\sum_{i=1}^{N} [w_i \overline{R}_i - T]^2 \rightarrow \ \text{min.} \tag{13}$$

Differentiating the squared sum of deviations with respect to corresponding weights, the following system of equations is obtained:

$$\begin{bmatrix} 2(\overline{R}_1)^2 & 0 & \dots. & \dots. & 0 & 0 \\ 0 & 2(\overline{R}_2)^2 & \dots. & \dots. & \text{symm} & 0 \\ \dots. & \dots. & \dots. & \dots. & \dots. & \dots. \\ \dots. & \dots. & \dots. & \dots. & \dots. & \dots. \\ 0 & \dots. & \dots. & \dots. & 2(\overline{R}_{N-1})^2 & 0 \\ 0 & 0 & \dots. & \dots. & 0 & 2(\overline{R}_N)^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ .. \\ \dots \\ \dots \\ .. \\ .. \\ w_{N-1} \\ w_N \end{bmatrix} = \begin{bmatrix} 2T\overline{R}_1 \\ 2T\overline{R}_2 \\ \dots. \\ \dots. \\ \dots. \\ 2T\overline{R}_{N-1} \\ 2T\overline{R}_N \end{bmatrix} \tag{14}$$

that facilitates the determination of values of the mixture-factor weights, w_i, and their envelope standard deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} [w_i \overline{R}_i - T]^2}{N-1}} \tag{15}$$

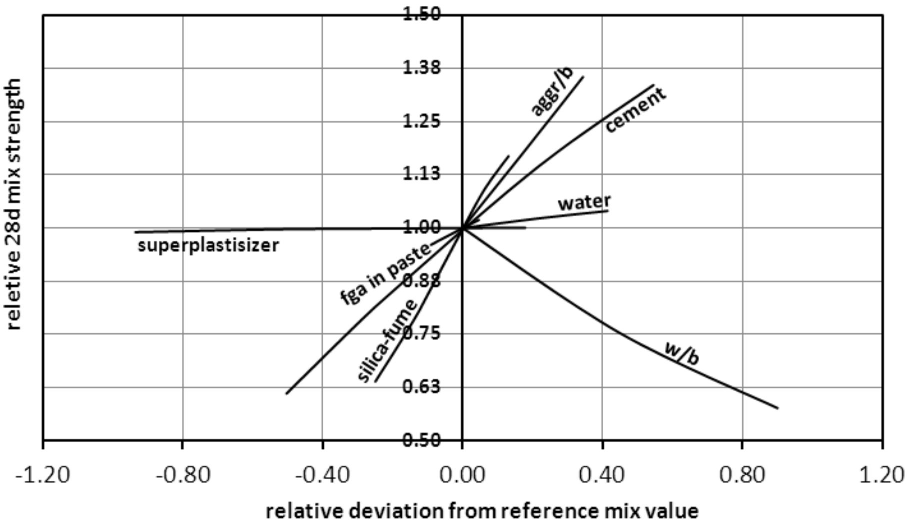

Fig. 3. Mixture-Strength Response Trace Plots of mixture L8

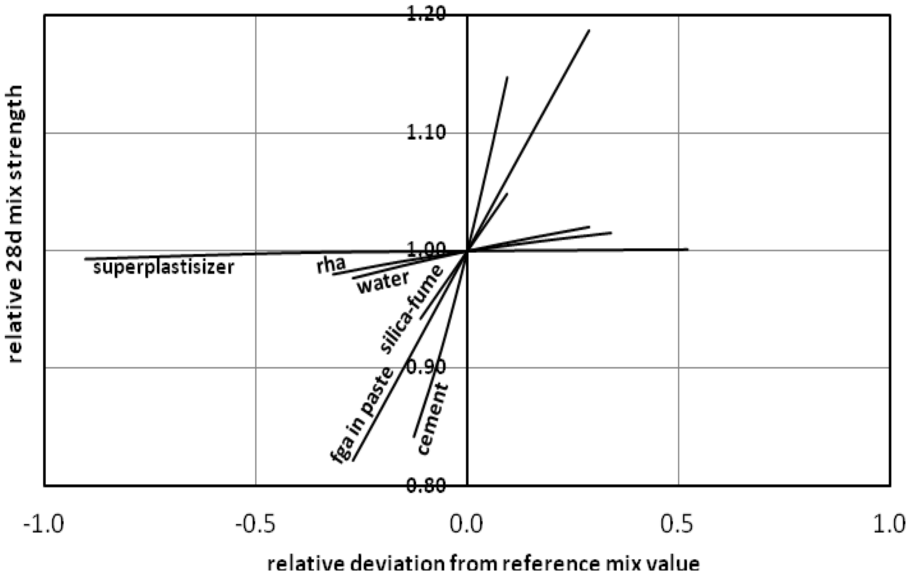

Fig. 4. Mixture-Strength Response Trace Plots of mixture RHA(5.6)

The constructed mixture-factor envelopes for concrete mixtures investigated in the study are presented in Fig. 6.

The constructed mixture-factor envelopes readily yield optimized mixture-compositions and corresponding optimized mixture-responses S_{target} and P_{target} by simply interpolating between upper and lower bound values of mixture-factor weights and mixture-factor values ($w_i^{UB}, w_i^{LB}, \pi_i^{UB}$ and π_i^{LB}) for a known mixture-response S_{ref} and P_{ref} of a known concrete mixture as follows:

For optimized mixture-compressive strength responses.

$$S_{target} = S_{ref}\left\{1 + \left\{\left[\frac{\pi_i - \pi_i^{LB}}{\pi_i^{UB} - \pi_i^{LB}}\right]\left[\frac{\sum_{i=1}^{N}\left|w_i^{UB} - w_i^{LB}\right|}{N}\right]\right\}\right\} \tag{16}$$

For optimized mixture-total porosity responses

$$P_{target} = P_{ref}\left\{1 - \left\{\left[\frac{\pi_i - \pi_i^{LB}}{\pi_i^{UB} - \pi_i^{LB}}\right]\left[\frac{N}{\sum_{i=1}^{N}\left|w_i^{UB} - w_i^{LB}\right|}\right]\right\}^3\right\} \tag{17}$$

Predicted optimized mixture-compositions and corresponding optimized mixture responses are summarized in Table 5.

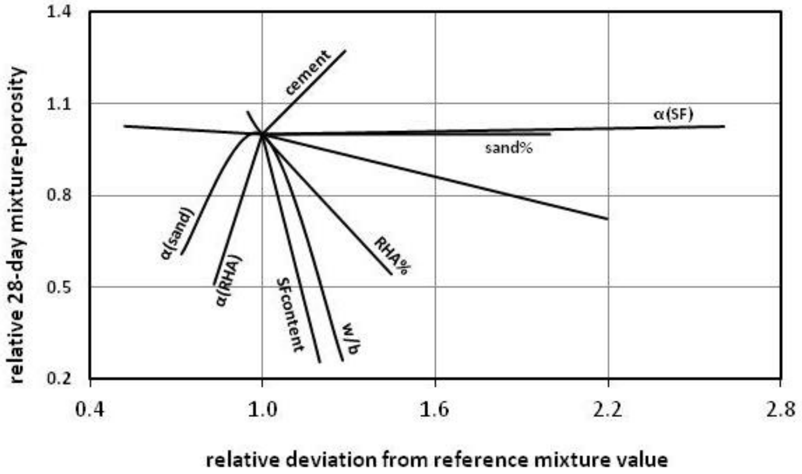

Fig. 5. Mixture-Porosity Response Trace Plots for mixture S7

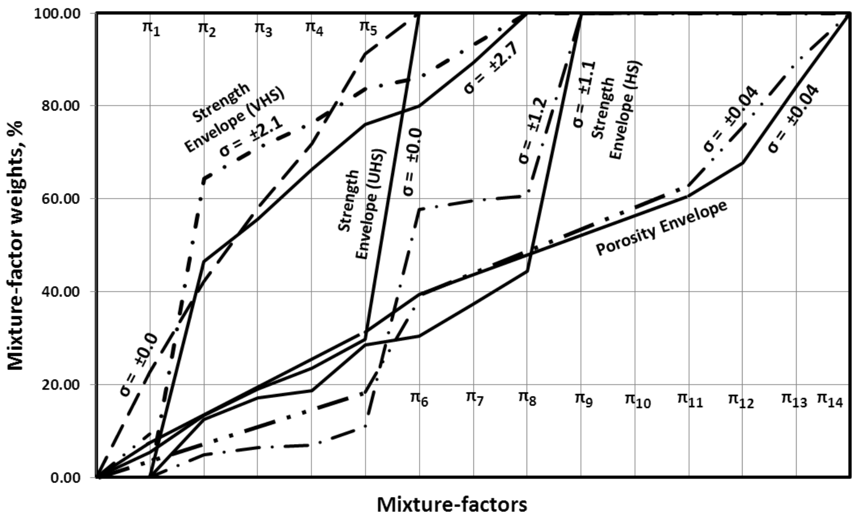

Fig. 6. Mixture-Factor Envelopes for Studied Mixture Properties

Table 5. Predicted Optimized Mixture Composition and Parameterized Mixture-Strength for HPC Mixtures

Reference Mixture ID	Optimized Mixtre factor	Sand %	$\frac{w}{b}$	C. aggr kg/cu. m	Water kg/cu. m	Cemnt kg/cu. m	FA kg/cu. m	SF kg/cu. m	SP kg/cu.m	RHA kg/cu.m	Entraind Air %	Slmp mm	PMS Model Result MPa	MFE Model Result MPa
MS5			0.219	647.1	150.4	541.9	82.2	62.7	15.4		1.8	267	81.3	87.1
MS8			0.230	627.5	153.8	525.1	82.3	62.3	12.2		1.8	241	81.6	88.1
MS18	FGA	54.5	0.228	651.2	150.4	513.1	82.4	62.9	11.0		2.2	254	78.7	83.0
T3			0.248	674.9	134.5	406.8	83.6	52.9	9.0		7.1	267	69.7	71.6
T11			0.340	643.0	182.6	401.3	82.6	52.7	3.9		4.8	114	53.9	58.0
T20			0.255	703.7	130.9	381.1	82.0	52.3	5.7		3.9	203	71.9	76.7
H2			0.285	1081.8	148.1	453.5	28.6	37.8	3.3				100.0	98.9
H4			0.245	1088.6	138.0	520.0		43.2	7.2				106.1	108.2
K14	C. aggr	37.5	0.213	1083.3	128.7	493.4		111.6	8.3			150	122.8	125.6
K15			0.210	1080.3	127.7	462.3		144.6	8.2				127.2	130.6
L5			0.250	1090.1	138.5	281.7	162.9	108.2	4.6				107.6	119.5
SF20				1140.0	171.0	950.0		190.0	8.7				183.4	190.3
SF10				1140.0	186.5	1036.4		103.6	8.7				171.2	185.8
RHA20(5.6)	C. aggr	100.0	0.18	1140.0	171.0	950.0		-	13.1	190.0		220	172.0	184.7
SF10RHA10 (5.6)				1140.0	171.0	950.0		95.0	13.1	95.0			177.9	188.1
K15	Cement	37.5	0.203	1139.1	116.1	495.0		75.7	8.2			150	128.8	133.3
L5	Cement		0.240	1131.0	126.5	303.0	175.2	27.5	4.7				118.1	114.8
MS8	Cement	54.5	0.222	654.6	146.3	537.3	56.3		9.8		1.9	241	79.7	87.3
SF10RHA10	Sf	100.0	0.18	1140.0	171.0	950.0		95.0	13.1	95.0		220	177.9	173.8
	Sp			1140.0	171.0	950.0		95.0	13.1	95.0			177.9	178.6
	Water		0.285	650.8	134.2	366.8	78.7	25.5	7.1		7.1		59.7	66.2
	Cement		0.244	646.1	123.0	400.4	77.7	25.2	7.0		7.0		65.7	66.5
T3	Sf	54.5	0.252	708.2	123.0	362.7	77.8	48.0	7.0		7.0	267	72.7	68.3
	Fa		0.263	606.4	112.5	314.9	52.9	22.2	2.5		1.5		67.8	63.6
	Sp		0.265	671.0	129.7	381.4	81.9	26.5	9.4		7.4		62.4	67.3

Results and Discussion

Results predicted by the proposed parameterized mixture-strength and mixture-porosity models are compared with mixture-test results and results of available models in Table 1, Table 2, Table 3 and Table 4. As can be observed, the proposed parameterized mixture-strength and mixture-porosity response models yield reasonably good results for most of the mixtures studied (with the following statistical performance metrics for an unbiased estimate of the prediction ability of the mixture-response models: mean absolute percentage error MAPE, normalized mean bias error NMBE and root mean square error RMSE values of 6.7%, -3.8% and 9.4, respectively).

The computed mixture-factor weights and constructed mixture-factor envelopes were used to predict optimized mixture-compositions, with mixture-factors limited to the ranges in the study domain, and these were in turn used to predict optimized mixture-performance(s) of interest. Optimization mixture-strength response results, presented in Table 5, indicate maximum increases of 8.7%, 12.6% and 12.6% in mixture-strength responses for mixture-optimization of higher strength, high strength and ultra-high strength HPC mixtures respectively and varying fractions of the same for mixture-factor optimization. Higher strength concrete mixtures have the least mixture-factor envelope-area (see Fig. 6) and hence respond the least to mixture-composition optimization efforts. Optimized mixture-strength responses predicted by the mixture-factor envelope (MFE) model compare favourably with those obtained by the parameterized mixture-strength (PMS) response model (with the following statistical performance metrics for an unbiased estimate of the prediction ability of the mixture-response models: mean absolute percentage error MAPE, normalized mean bias error NMBE and root mean square error RMSE values of 7.6%, -3.7% and 6.5, respectively) and although the results for the parameterized mixture-porosity (PMP) model are not tabulated it has a lean mixture-factor envelope-area (see Fig. 6) and hence responds the least to mixture-porosity based mixture-composition optimization efforts.

The optimal values of mixture-factors for fly-ash and silica-fume yield more strength-efficient T3 mixture while optimizing UHS HPC mixtures through graded aggregates yielded strength-efficient and cement-efficient mixtures, resulting, potentially, in significant cost saving in the mixture production.

HPC mixtures can be similarly optimized for workability (through the mix-slump factor) and durability (through the mix entrained-air-content factor). Attempts at modelling optimization of HPC mixture-compositions in the larger mixture-modelling research community—even with the non-traditional advanced machine learning optimization approaches like sequential learning neural network (SLNN) or neuro-fuzzy computing techniques such as the adaptive neuro-fuzzy inference system (ANFIS)—have thus far only yielded, at best, qualitative characterizations of concrete performance or mixture optimization [19, 21].

It is acknowledged that these models are derived from laboratory-mixture test responses and therefore their application to field cement concrete mixtures (in-situ concretes) suspect or uncertain. Experience has, however, shown that a high percentage (up to 90%) of laboratory-mixture test responses (in-situ concrete) in-situ concretes under good field practices [22]. PCs have already moved from laboratory research to practical applications and already occupy a sizeable share of the market although most of these applications have been limited to proprietary blends and non-in situ construction (commercial ready mix products and pre-cast applications) and even convenience blends but are in general more expensive (by an order of upwards of twenty) than non-proprietary conventional cement concretes mainly owing to proprietary specifications of mixture proportions of non-proprietary HPC mixtures usually being based on trial and error methods than any settled material/behavioural laws or some quantitative characterization of its performance [23, 24, 25].

Conclusions

The study has offered a quantitative characterization of the performance of HPC mixtures that allows optimal tailoring of mixture requirements to mixture performance of HPCs by explicitly relating performance (user) specifications to mixture (producer) requirements and make possible optimized trade-offs between them where the three main performance specifications—strength, workability, and durability variously specified (the strength, through the desired compressive strength; workability, through the desired slump; and durability, through some given exposure condition)—can be explicitly achieved through variously specified mixture requirements (compressive strength, via some water/cement ratio; workability, via some indication of water content per unit volume of concrete; and durability, via some indication of some minimum cement content and maximum water/cement ratio). The study results suggest the performance of a known mixture (its strength, workability, and durability) can be improved by a determinable amount and an optimized mixture-composition reliably determined through mixture-factor envelopes largely by increasing the binder content of the mixture and/or the graded aggregate content of the mixture. The described approach to HPC mixture design was reliably used to optimally modify or customize available HPC formulations and offered a valuable tool for minimizing the number of trial batches needed to identify the optimal mixture-factors for achieving the desired economy and efficiency while at the same time achieving the desired performance.

List of Abbreviations,

The following symbols are used in this paper:

N	-	the total number of mixtures
S_{ref}	-	mixture-strength response of a known reference mixture
S_{target}	-	mixture-strength targeted optimized response of a mixture
P_{ref}	-	mixture-porosity response of a known reference mixture
P_{target}	-	mixture-porosity targeted optimized response of a mixture
R_i	-	linearly weighted summated response function of mixture
$\overline{R}_i$	-	individual normalized response function of a mixture
T	-	the target value for the response functions facilitates the determination of values of the weights
$f_i\,(a)$	-	maxima (or minima) of response trace-plot function
$f_{ref}(a)$	-	maxima (or minima) of response trace-plot function for a reference mixture
w_i	-	the weight for individual normalized response function of a mixture
$w^{UB}{}_i$	-	upper bound weight-value for individual normalized response function i
$w^{LB}{}_i$	-	lower bound weight-value for individual normalized response function i
$\pi^{UB}{}_i$	-	upper bound mixture-factor value i
$\pi^{LB}{}_i$	-	lower bound mixture-factor value i
σ	-	envelope standard deviation
α_{cement}	-	packing density of cement
α_{sand}	-	packing density of sand
α_{RHA}	-	packing density of RHA
$\alpha_{silica\text{-}fume}$	-	packing density of silica-fume

Acknowledgements

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors. However, the author thanks The Department of Civil Engineering at The Polytechnic for the resources availed him towards the production of the manuscript and also recognize the helpful comments of several colleagues in the department.

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Received: January 6, 2018
Accepted: February10, 2018